Bharat
BoudhIKS

Volume-6

Vedic Mathematics

Wisdom Beyond Time

Eternal Guidance : Youth Excellence

A Concise Book for IKS based Competitive Exams (NET and UPSC)

Introduction

Tracing the Timeless Genius of Bharatiya Mathematics

Mathematics in Bharat is not merely a record of numbers and equations-it is a living tradition that stretches back over 5,000 years. From the brick ratios of the Indus Valley Civilization to the infinite series of the Kerala School, the Bharatiya subcontinent has cultivated one of the most profound and continuous mathematical traditions in the world.

1. Beginnings in the Indus Valley (3500–1500 BCE): Geometry Before Scripts

Long before the rise of formal scripts, the people of the Indus Valley were already applying mathematical principles in their daily lives. Archaeological excavations at Harappa and Mohenjo-daro reveal an astonishing command over standardized weights and measures, geometry in urban planning, and engineering precision-evident in their famous **"Indus inch"** rulers accurate to 0.005 inches. Their bricks followed the 4:2:1 ratio, still considered ideal today, and their city layouts show an advanced understanding of symmetry, grids, and spatial efficiency.

2. Vedic Era and the Sulba Sūtras (1500–500 BCE): Geometry Meets Ritual

The earliest **written** mathematical texts of Bharat emerge not in isolation, but embedded in spiritual practice. The **Sulba Sūtras,** appendices to the **Kalpa Vedāṅga,** are manuals for constructing complex Vedic fire altars. Composed between 800 and 500 BCE, these terse aphoristic texts by sages like **Baudhāyana, Āpastamba, Kātyāyana,** and **Mānava** reveal an astonishing depth of mathematical insight:

- A statement of the **Pythagorean theorem**-centuries before Pythagoras.
- An approximation of √2 accurate to five decimal places.
- Geometric transformations between squares and circles.
- Early concepts of irrational numbers and Pythagorean triples.

These texts serve as the philosophical and practical foundation of Bharatiya mathematical thought-where mathematics served both ritual and reason.

3. Vedic Mathematics and the Vedāṅgas: Math in the Service of Knowledge

Mathematics in ancient Bharat was not compartmentalized-it was the beating heart of all disciplines. Among the six **Vedāṅgas** (limbs of the Vedas), three in particular-**Jyotiṣa** (astronomy), **Chandas** (prosody), and **Kalpa** (ritual)-relied heavily on mathematical techniques. In **Vedāṅga Jyotiṣa**, attributed to **Lagadha** (c. 1400 BCE), we find astronomy and timekeeping bound deeply with numeracy and combinatorics.

This integration reflects the Bharatiya worldview: mathematics was not a tool, but a **jewel**-*"the crest of a peacock, the gem on a serpent's hood,"* as the ancients proclaimed.

4. Jain Contributions (400 BCE – 400 CE): Infinity and the Abstract Mind

While Vedic traditions were deeply spiritual and ritualistic, the **Jain mathematicians** approached mathematics with a unique clarity and abstraction. They were the **first to conceptualize and name 'zero' as 'śūnya'**, developed early ideas of **factorials and permutations**, and classified **infinities into five different types**. Their emphasis on enumeration, vast numerical systems, and logical classification laid the groundwork for later algebraic innovations.

Important Jain texts like the *Sūrya Prajñapti* and *Sthānāṅga Sūtra* represent a pivotal shift in Bharatiya mathematics: from altar geometry to pure mathematical contemplation.

5. Classical Bharatiya Mathematics (400–1200 CE): The Golden Age

The classical period marks the crystallization of Bharatiya mathematics into formal, structured treatises. The two most towering figures of this era are:

- **Āryabhaṭa-I** (476–550 CE), whose *Āryabhaṭīya* introduced:
  - The **decimal system** with zero as a placeholder,
  - Trigonometric functions like sine and cosine,
  - Accurate calculation of **π (pi)** to 3.1416,
  - Techniques for **algebra and planetary motion**.
- **Brahmagupta** (598–668 CE), in his *Brāhmasphuṭasiddhānta*, gave:
  - Rules for **operations with zero**,
  - Treatment of **negative numbers**,
  - Solutions to **quadratic equations**,
  - Geometry of **cyclic quadrilaterals**,
  - Precise **astronomical computations**.

Their work, rooted in Sanskrit verse and prose, influenced not only Bharat but also the Islamic and later the Western mathematical world.

6. The Bakhshali Manuscript (700–1100 CE): A Hidden Treasure

Discovered in 1881 and written on birch bark, the **Bakhshali Manuscript** is now recognized as the **earliest surviving manuscript of Bharatiya mathematics**. Although its exact date is debated, its content bridges multiple periods. It presents:

- Operations with **fractions and square roots**,
- Use of **progressions and algebraic equations**,
- An early symbol for **zero**-a dot.

It offers valuable insights into the pedagogy and practice of mathematics in ancient Bharatiya schools.

7. Kerala School (14th–16th Century): Calculus Before Newton

In the temple towns of Kerala, a group of visionary scholars, starting with **Mādhava of Sangamagrama**, pioneered what many now recognize as **proto-calculus**:

- **Infinite series** for sine, cosine, and arctangent,
- **Pi** calculated using multiple series approximations,
- **Power series** expansion techniques,

- The monumental text *Yuktibhāṣā* by **Jyeṣṭhadeva** is often dubbed the **first calculus textbook**.

These scholars anticipated methods of Newton and Leibniz by centuries, though their work remained less known globally due to limited transmission.

8. Transmission to the World: A Legacy That Travelled

Bharatiya mathematical texts and concepts spread westward through:

- **Arabic translations** of texts like *Āryabhaṭīya* and *Siddhāntas* during the Islamic Golden Age,
- **Trade routes and scholarly exchanges** from Nalanda to Baghdad,
- **Adoption of the Hindu-Arabic numeral system**, including the revolutionary concept of zero and positional decimal notation.

This transmission laid the foundations for modern global mathematics.

9. The Reawakening: What is "Vedic" Mathematics?

In the early 20th century, **Jagadguru Śrī Bhāratī Kṛṣṇa Tīrtha**, a revered scholar and Śaṅkarācārya of Puri, undertook deep meditative study of the **Atharva Veda** and its appendices. Between 1911–1918, he claimed to "rediscover" 16 **mathematical sūtras** and 13 **sub-sūtras**-aphoristic techniques that could be applied to arithmetic, algebra, geometry, and even calculus.

Though not traceable directly in the Vedas, these sūtras-later published posthumously in **1965**-formed the core of what we today know as **Vedic Mathematics**: a system of **mental mathematics** that is elegant, efficient, and rooted in pattern recognition and simplicity.

10. Interpreting the "Vedic" Label

Scholars have debated the historical veracity of these rediscovered sutras. While the exact formulations are absent from Vedic manuscripts, many believe they draw inspiration from the mental agility, pattern-based methods, and cryptic aphorisms seen in ancient Bharatiya mathematical traditions.

Thus, Vedic Mathematics is not a direct continuation but a **20th-century systematization**-a spiritual and intellectual homage to Bharat's ancient mathematical legacy.

11. Mathematics as a Way of Thinking

Throughout Bharatiya history, mathematics was not simply about solving equations. It was a way to **understand the cosmos, structure poetic meter, construct sacred space**, and **explore the infinite**. From the rhythm of Vedic chants to the logic of Jain thinkers, from the planetary orbits of Āryabhaṭa to the series expansions of Kerala mathematicians, it was an exploration of **order, harmony, and truth**.

The tradition of Bharatiya mathematics is not merely a tale of counting numbers - it is a profound journey of consciousness and understanding the cosmos. From the silence of zero to the vastness of infinity, from the sacred geometry of the yajña vedi (ritual altar) to the precise motions of celestial bodies, this legacy reflects a harmonious blend of **continuity and innovation**.

In this tradition, mathematics was not just a tool for measurement, but a means of **realizing knowledge itself**.

As expressed in the ancient texts:

> **"गणितं मूर्धनि स्थितं, शास्त्राणां गणकं स्मृतम्।"**
> **"Gaṇitaṁ mūrdhani sthitaṁ, śāstrāṇāṁ gaṇakaṁ smṛtam."**
>
> *- Līlāvatī, Bhāskarācārya*

Meaning: (Mathematics stands at the head; it is regarded as the calculator of all sciences.)

> **"सर्वं गणितमेवात्र, नास्ति किंचित् गणनं विना।"**
> **"Sarvaṁ gaṇitamevātra, nāsti kiñcit gaṇanaṁ vinā."**
>
> *- Siddhāntaśiromaṇi, Bhāskarācārya*

Meaning: (Everything here is mathematics; nothing exists without calculation.)

> **"यत्र यत्र गणितं तत्र तत्र ज्योतिर्मयं जगत्।"**
> **"Yatra yatra gaṇitaṁ, tatra tatra jyotirmayaṁ jagat."**
>
> *- Tattvasaṅgraha*

Meaning: (Wherever there is mathematics, there is the radiant presence of the cosmos.)

Thus, as we walk through each chapter of this journey, we are not merely exploring techniques and equations, but witnessing the vision of the sages - who saw **truth through numbers**, and **order in the infinite**.

Let us always remember this Vedic sentiment:

गणितं शास्त्रस्य शिरः।"

("Ganitam Shastrasya Sirah.")

Meaning: (Mathematics is the crown of all knowledge systems.)

May this journey rekindle not only intellectual curiosity but also a deep reverence for the mathematical spirit of ancient Bharat – a legacy that continues to inspire, enlighten, and guide.

Chapter 1

Duplex Method: A General Method for Squaring of Numbers

The duplex method is a technique used for finding the square of a number. The first step of squaring of numbers using Duplex method involves finding duplex of numbers. The duplex (D) is a number associated with a number calculated as below. Method of calculating Duplex varies with number of digits.

1. For a single digit (a), the duplex is simply its square:

$$D(a) = a^2$$

2. For a two-digit number (ab), the duplex is calculated as:

$$D(ab) = 2 \times a \times b$$

3. For a three-digit number (abc), the duplex is given by:

$$D(abc) = 2 \times a \times c + b^2$$

4. For a four-digit number ($abcd$), the duplex is:

$$D(abcd) = 2 \times a \times d + 2 \times b \times c$$

5. For a five-digit number ($abcde$), the duplex is:

$$D(abcde) = 2 \times a \times e + 2 \times b \times d + c^2$$

6. The pattern continues for larger numbers, where we consider symmetrical products and the middle term squared when applicable.

Next we explain with help of examples, how duplex can be used to find squares of numbers.

Example 1: Squaring 23

We compute the duplexes:

$$D(2) = 2^2 = 4$$

$$D(23) = 2 \times 2 \times 3 = 12$$

$$D(3) = 3^2 = 9$$

Arranging these in sequence:

$$D(2) / D(23) / D(3) = 4 / 12 / 9$$

Adjusting the carries from right to left (Keeping single digit at each place)

$$4 + 1 / 2 / 9 \Rightarrow 529$$

Example 2: Squaring 89

Step 1: Compute duplexes

$$D(8) = 8^2 = 64$$

$$D(89) = 2 \times 8 \times 9 = 144$$

$$D(9) = 9^2 = 81$$

Step 2: Arrange duplex values

$$64 / 144 / 81$$

Step 3: Adjust the carries

$$64 \ / \ 144 + 8 \ / \ 1$$

$$= 64 + 15 \ / \ 2 \ / \ 1$$

$$= 7921$$

Example 3: Squaring 314

Step 1: Compute duplexes

$D(3) = 9$

$D(31) = 2 \times 3 \times 1 = 6$

$D(314) = 2 \times 3 \times 4 + 1^2 = 24 + 1 = 25$

$D(14) = 2 \times 1 \times 4 = 8$

$D(4) = 16$

Step 2: Sequence of duplex values

$$9 / 6 / 25 / 8 / 16$$

Step 3: Adjust carries from right to left

$$9 / (6 + 2) / 5 / (8 + 1) / 6 = 98596$$

Example 4: Squaring 1468

Step 1: Compute all necessary duplexes

$D(1) = 1$

$D(14) = 2 \times 1 \times 4 = 8$

$D(146) = 2 \times 1 \times 6 + 4^2 = 12 + 16 = 28$

$D(1468) = 2 \times 1 \times 8 + 2 \times 4 \times 6 = 16 + 48 = 64$

$D(468) = 2 \times 4 \times 8 + 62 = 64 + 36 = 100$

$D(68) = 2 \times 6 \times 8 = 96$

$D(8) = 64$

Step 2: Sequence of values

$$1 / 8 / 28 / 64 / 100 / 96 / 64$$

Step 3: Adjust carries step-by-step

Rightmost: $64 \Rightarrow 4$, carry 6

$96 + 6 = 102 \Rightarrow 2$, carry 10

$100 + 10 = 110 \Rightarrow 0$, carry 11

$64 + 11 = 75 \Rightarrow 5$, carry 7

$28 + 7 = 35 \Rightarrow 5$, carry 3

$8 + 3 = 11 \Rightarrow 1$, carry 1

$1 + 1 = 2$

Final Result

$$2155024$$

Practice Exercises

Try squaring the following numbers using the Duplex Method:

1. 56
2. 67
3. 78
4. 135
5. 246
6. 357
7. 489
8. 5689
9. 1234
10. 357

Why Use the Duplex Method?

The Duplex Method is especially helpful when dealing with multi-digit numbers. By transforming a large square into a sequence of small, symmetric calculations, it reduces computational complexity and enhances mental math skills. With regular practice, this method becomes a fast and reliable technique for squaring numbers of any size.

Chapter 2

Squaring Numbers Ending in 5

Vedic Mathematics offers a remarkably efficient method for squaring numbers that end in the digit 5. This technique simplifies what might otherwise be a lengthy multiplication process and is especially useful for mental calculations.

General Method

To square a number ending in 5, follow these steps:

1. Remove the digit 5 from the end of the number. Let the remaining part be a.
2. Compute the product $a(a + 1)$.
3. Append the digits "25" to the result obtained in Step 2.

The final number is the square of the original number. Let us explore this method through several examples.

Example 1: Find 75^2

Step 1: Remove the last digit 5. We are left with:

$$a = 7$$

Step 2: Multiply a with its successor:

$$7 \times 8 = 56$$

Step 3: Append "25" to the result:

$$75^2 = 5625$$

Conclusion: The square of 75 is 5625.

Example 2: Find 115^2

Step 1: Remove the last digit 5.

$$a = 11$$

Step 2: Compute the product:

$$11 \times 12 = 132$$

Step 3: Append "25" to obtain:

$$115^2 = 13225$$

Conclusion: The square of 115 is 13225.

Example 3: Find 1255^2

This is a larger number, but the method remains the same. Here, an additional layer of computation is required.

Step 1: Remove the last digit 5.

$$a = 125$$

We now compute 125×126, which can be written as:

$$125 \times 126 = 125^2 + 125$$

Since 125 also ends in 5, we apply the same technique to find 125^2.

Substep 1: Remove the digit 5:

$$a = 12$$

Substep 2: Multiply with successor:

$$12 \times 13 = 156$$

Substep 3: Append "25":

Now, compute:

$$125^2 = 15625$$

$$125 \times 126 = 125^2 + 125 = 15625 + 125 = 15750$$

Finally, append "25":

$$1255^2 = 1575025$$

Conclusion: The square of 1255 is 1575025.

Example 4: Find 1555^2

This is a larger number, but the method remains the same. Here, a additional layer of computation is required.

Step 1: Remove the last digit 5.

$$a = 155$$

We now compute 155 × 156, which can be written as:

$$155 \times 156 = 155^2 + 155$$

Since 155 also ends in 5, we apply the same technique to find 155^2.

Substep 1: Remove the digit 5:

$$a = 15$$

Substep 2: Multiply with successor:

$$15 \times 16 = 240$$

Substep 3: Append "25":

Now, compute:

$$155^2 = 24025$$

$$155 \times 156 = 155^2 + 155 = 24025 + 155 = 24180$$

Finally, append "25":

$$1555^2 = 2418025$$

Conclusion: The square of 1555 is 2418025.

Example 5: Find 3255^2

This is a larger number, but the method remains the same. Here, an additional layer of computation is required.

Step 1: Remove the last digit 5.

$$a = 325$$

We now compute 325 × 326, which can be written as:

$$325 \times 326 = 325^2 + 325$$

Since 325 also ends in 5, we apply the same technique to find 325^2.

Substep 1: Remove the digit 5:

$$a = 32$$

Substep 2: Multiply with successor:

$$32 \times 33 = 1056$$

Substep 3: Append "25":

$$325^2 = 105625$$

Now, compute:

$$325 \times 326 = 325^2 + 325 = 105625 + 325 = 105950$$

Finally, append "25":

$$3255^2 = 10595025$$

Conclusion: The square of 3255 is 10595025.

Why the Method Works

This technique is based on the algebraic identity:

$$(10a + 5)^2 = 100a(a + 1) + 25$$

Here, the expression $a(a + 1)$ generates the digits before the final "25", which always appears as the square of 5. The efficiency and elegance of this method lie in its ability to decompose a large computation into manageable parts.

Exercises

Use the above method to find the square of each of the following numbers:

1. 15
2. 95
3. 65
4. 205
5. 165
6. 145
7. 105
8. 1055
9. 7555
10. 1155

Chapter 3

Squaring Numbers Ending in 1

Vedic Mathematics provides an elegant technique for quickly squaring numbers that end in the digit 1. This method breaks the process into manageable steps, making it ideal for mental math and enhancing numerical intuition.

General Method

To find the square of any number ending in 1, follow these steps:

1. Exclude the last digit 1. Let the remaining part of the number be denoted by a.
2. Compute $2 \times a$.
3. Compute a^2.
4. Combine the results in the format:

$$a^2 / (2 \times a) / 1$$

Then, adjust the carry from right to left — keeping one digit at each place — to arrive at the final result.

This process may seem unusual at first, but once practiced, it becomes a rapid and reliable mental shortcut.

Example 1: Find 61^2

Step 1: Remove the last digit 1:

$$a = 6$$

Step 2: Compute $2 \times a$:

$$2 \times 6 = 12$$

Step 3: Compute a^2:

$$6^2 = 36$$

Step 4: Combine the values as:

$$36 \; / \; 12 \; / \; 1$$

Next we adjust the carries.

Rightmost digit: 1 (write 1, carry 0)
Next: 12 + carry = 12 (write 2, carry 1)
Now: 36 + 1 = 37

So the final answer is: 3721

Example 2: Find 91^2

Step 1: Remove the last digit 1:

$$a = 9$$

Step 2: Compute $2 \times a = 18$

Step 3: Compute $a^2 = 81$

Step 4: Combine the results:

$$81 \; / \; 18 \; / \; 1$$

Adjusting the digits from right to left:

Rightmost digit: 1 (write 1, carry 0)
Next: 18 + carry = 18 (write 8, carry 1)
Now: 81 + 1 = 82

So the final answer is: 8281

Example 3: Find 121^2

Step 1: Remove the digit 1:

$$a = 12$$

Step 2: Compute $2 \times a = 24$

Step 3: Compute $a^2 = 144$

Step 4: Combine as:

$$144 \; / \; 24 \; / \; 1$$

Perform the right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)
Next: 24 + carry = 24 (write 4, carry 2)
Now: 144 + 2 = 146

So the final answer is: 14641

Example 4: Find 211^2

Step 1: Remove the digit 1:

$$a = 21$$

Step 2: Compute $2 \times a = 42$

Step 3: Compute $a^2 = 441$

Step 4: Combine as:

$$441 \; / \; 42 \; / \; 1$$

Perform the right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)
Next: 42 + carry = 42 (write 2, carry 4)
Now: 441 + 4 = 445
So the final answer is: 44521

Example 5: Find 1511^2

Step 1: Remove the digit 1:

$$a = 151$$

Step 2: Compute $2 \times a = 302$

Step 3: Compute $a^2 = 151^2$

Since 151 ends with 1, we apply the same method to comput 151^2 first.

Substep 1: Remove the digit 1:

$$a = 15$$

Substep 2: Compute $2 \times a = 30$

Substep 3: Compute $a^2 = 225$

Substep 4: Combine as:

$$225 \; / \; 30 \; / \; 1$$

Perform the right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)
Next: 30 + carry = 30 (write 0, carry 3)
Now: 225 + 3 = 228
So, $151^2 = 22801$

Now return to our original calculation:

Step 4: Combine as:

$$22801 \; / \; 302 \; / \; 1$$

Perform the right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)
Next: 302 + carry = 302 (write 2, carry 30)
Now: 22801 + 30 = 22831
So the final answer is: 2283121

Example 6: Find 4511^2

Step 1: Remove the digit 1:

$$a = 451$$

Step 2: Compute $2 \times a = 902$

Step 3: Compute $a^2 = 451^2$

Since 451 ends with 1, we apply the same method to compute 451^2 first.

Substep 1: Remove the digit 1:

$$a = 45$$

Substep 2: Compute $2 \times a = 90$

Substep 3: Compute $a^2 = 2025$

Substep 4: Combine as:

$$2025 \; / \; 90 \; / \; 1$$

Perform the right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)
Next: 90 + carry = 90 (write 0, carry 9)
Now: 2025 + 9 = 2034

So,

$$451^2 = 203401$$

Now return to our original calculation:

Step 4: Combine as:

$$203401 \; / \; 902 \; / \; 1$$

Perform the right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)
Next: 902 + carry = 902 (write 2, carry 90)
Now: 203401 + 90 = 203491
So the final answer is: 20349121

Example 7: Find 12511^2

Step 1: Remove the digit 1:

$$a = 1251$$

Step 2: Compute $2 \times a = 2502$

Step 3: Compute $a^2 = 1251^2$

Since 1251 ends in 1, we apply the same method again.

Nested Calculation for 1251^2:

Substep 1: Remove the digit 1:

$$a = 125$$

Substep 2: Compute $2 \times a = 250$

Substep 3: Compute $a^2 = 125^2$

Since 125 ends in 5, we apply the "ends in 5" trick:

Subsubstep 1: Remove 5:

$$a = 12$$

Subsubstep 2: Multiply with successor:

$$12 \times 13 = 156$$

Subsubstep 3: Append "25":

$$125^2 = 15625$$

Now back to 1251^2:

Substep 4: Combine as:

$$15625 \ / \ 250 \ / \ 1$$

Right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)

Next: 250 + carry = 250 (write 0, carry 25)

Now: 15625 + 25 = 15650

So, $1251^2 = 1565001$

Now go back to the main number 12511:

Step 4: Combine as:

$$1565001 \ / \ 2502 \ / \ 1$$

Final right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)

Next: 2502 + carry = 2502 (write 2, carry 250)

Now: 1565001 + 250 = 1565251

So the final answer is: 156525121

Example 8: Find 1651511^2

Step 1: Remove the digit 1:

$$a = 165151$$

Step 2: Compute $2 \times a = 330302$

Step 3: Compute $a^2 = 165151^2$

Since 165151 ends in 1, we apply the same method again.

Next we calculate 165151^2:

Substep 1: Remove the digit 1:

$$a = 16515$$

Substep 2: Compute $2 \times a = 33030$

Substep 3: Compute $a^2 = 16515^2$

Since 16515 ends in 5, we apply the "ends in 5" trick.

Now we calculate 16515^2:

Subsubstep 1: Remove the digit 5:

$$a = 1651$$

Subsubstep 2: Multiply with successor:

$$1651 \times 1652$$

Now, compute 1651^2 using the "ends in 1" method again.

Again we calculate 1651^2:

Step 1: Remove the digit 1:

$$a = 165$$

Step 2: $2 \times a = 330$

Step 3: Compute 165^2

Since 165 ends in 5, we use the same trick:

Step 1: Remove 5:

$$a = 16$$

Step 2: $16 \times 17 = 272$

Step 3: Append "25":

$$165^2 = 27225$$

Now continue with 1651^2:

Step 4: Combine as:

27225 / 330 / 1

Right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)

Next: 330 + carry = 330 (write 0, carry 33)

Now: 27225 + 33 = 27258

So,

$$1651^2 = 2725801$$

Now back to compute:

$$1651 \times 1652 = 1651^2 + 1651 = 2725801 + 1651 = 2727452$$

Then,

$$16515^2 = 272745225$$

Now go back to 165151^2:

Step 4: Combine as:

272745225 / 33030 / 1

Right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)

Next: 33030 + carry = 33030 (write 0, carry 3303)

Now: 272745225 + 3303 = 272748528

So,

$$165151^2 = 27274852801$$

Now, back to our original number:

Step 4: Combine as:

27274852801 / 330302 / 1

Final right-to-left adjustment:

Rightmost digit: 1 (write 1, carry 0)

Next: 330302 + carry = 330302(write 2, carry 33030)

Now: 27274852801 + 33030 = 27274885831

So the final answer is: 2727488583121

Conclusion

This method demonstrates how complex multiplication can be simplified using simple algebraic manipulation and clever arrangement. The key lies in breaking down the number and reconstructing it with careful carry-over adjustments.

Try it Yourself!

Use the same method to compute the squares of the following numbers:

1. 31
2. 41
3. 81
4. 101
5. 131
6. 1351
7. 711
8. 715
9. 7151
10. 12155
11. 1751151

Chapter 4

Urdhva Tiryagbhyam Method for Multiplication

Urdhva Tiryagbhyam is a Vedic mathematics sutra used for multiplication. It means "Ver- tically and Crosswise" and provides a systematic method to multiply numbers efficiently.

Method for multiplication of two digit numbers

- Step 1. Multiply vertically unit place digits, say *a*.
- Step 2. Multiply crosswise and add, say *b*.
- Step 3. Multiply vertically second place digits, say *c*.
- Step 4. The required multiplication is obtained by adjusting the carry from right to left, by keeping one digit in its place in the expression, *c* / *b* / *a*.

Next, we illustrate the above method with the help of some examples.

Example 1: Multiplication of 76 by 42

Step 1. Multiply unit digits: $6 \times 2 = 12$

Step 2. Crosswise multiplication and addition: $(7 \times 2) + (6 \times 4) = 14 + 24 = 38$

Step 3. Multiply second place digits: $7 \times 4 = 28$

Step 4. So 76×42 is obtained by adjusting the carry from right to left in expression 28/ 38 /12.

So final answer is, $76 \times 42 = 3192$

Example 2: Multiplication of 58 by 34

Step 1: Multiply unit digits: $8 \times 4 = 32$

Step 2: Crosswise multiplication and addition: $(5 \times 4) + (8 \times 3) = 20 + 24 = 44$

Step 3: Multiply second place digits: $5 \times 3 = 15$

So 58×34 is obtained by adjusting the carry from right to left in expression 15/ 44 /32.

So, $58 \times 34 = 1972$

Example 3: Multiplication of 76 by 32

Step 1: Multiply unit digits: $6 \times 2 = 12$

Step 2: Crosswise multiplication and addition: $(7 \times 2) + (6 \times 3) = 14 + 18 = 32$

Step 3: Multiply second place digits: $7 \times 3 = 21$

So 76×32 is obtained by adjusting the carry from right to left in expression 21/ 32 /12.

Start from the right: 12 (write 2, carry 1)

$32 + 1 = 33$ (write 3, carry 3)

$21 + 3 = 24$

So the final result is: **2432**

Therefore, $76 \times 32 = 2432$

Method for multiplication of three digit numbers

The same vertical and crosswise method can be extended to three-digit numbers. The process involves:

Step 1: Multiply the rightmost digits, say result is *a*.

Step 2: Multiply crosswise for the last two digits and sum the results, say sum is *b*.

Step 3: Multiply crosswise for all three digits and sum, say sum is *c*.

Step 4: Multiply crosswise for the first two digits and sum, say sum is *d*.

Step 5: Multiply the leftmost digits, say result is *e*.

Step 6: The required multiplication is obtained by adjusting the carry from right to left, by keeping one digit in its place in the expression, *e* /*d* /*c* / *b* / *a*.

Example 4: Multiplication of 566 by 281

Step 1: Multiply unit digits: $6 \times 1 = 6$

Step 2: Cross multiply of right most two digits and sum: $(6 \times 8) + (6 \times 1) = 48 + 6 = 54$

Step 3: Cross multiply all three digits and sum :
(5 × 1) + (6 × 8) + (6 × 2) = 5 + 48 + 12 = 65
Step 4: Cross multiply of left most two digits and sum:
(5 × 8) + (6 × 2) = 40 + 12 = 52
Step 5: Multiply leftmost digits: 5 × 2 = 10
Step 6: So 566 × 281 is obtained by adjusting the carry from right to left in expression 10 /52 /65 / 54 / 6.
So, 566 × 281 = 159046

Example 5: Multiplication of 472 by 342

Step 1: Multiply unit digits: 2 × 2 = 4
Step 2: Cross multiply of rightmost two digits and sum:
(7 × 2) + (2 × 4) = 14 + 8 = 22
Step 3: Cross multiply all three digits and sum:
(4 × 2) + (7 × 4) + (2 × 3) = 8 + 28 + 6 = 42
Step 4: Cross multiply of left two digits and sum:
(4 × 4) + (7 × 3) = 16 + 21 = 37
Step 5: Multiply leftmost digits: 4 × 3 = 12
Step 6: So 472 × 342 is obtained by adjusting the carry from right to left in expression 12 / 37 / 42 / 22 / 4.
Start from the right: 4 (write 4)
22 (write 2, carry 2)
42 + 2 = 44 (write 4, carry 4)
37 + 4 = 41 (write 1, carry 4)
12 + 4 = 16
So the final result is: **161424**
Therefore, 472 × 342 = 161424.

Example 6: Multiplication of 711 by 582

Step 1: Multiply unit digits: 1 × 2 = 2
Step 2: Cross multiply of rightmost two digits and sum:
(1 × 2) + (1 × 8) = 2 + 8 = 10
Step 3: Cross multiply all three digits and sum:
(7 × 2) + (1 × 8) + (1 × 5) = 14 + 8 + 5 = 27
Step 4: Cross multiply of left two digits and sum:

(7 × 8) + (1 × 5) = 56 + 5 = 61
Step 5: Multiply leftmost digits: 7 × 5 = 35
Step 6: So 711 × 582 is obtained by adjusting the carry from right to left in expression 35 / 61 / 27 / 10 / 2.
Start from the right: 2 (write 2) 10 (write 0, carry 1)
27 + 1 = 28 (write 8, carry 2)
61 + 2 = 63 (write 3, carry 6)
35 + 6 = 41
So the final result is: **413802**
Therefore, 711 × 582 = 413802

Exercises

Solve the following using the Urdhva Tiryagbhyam method:
Step 1: 47 × 36
Step 2: 89 × 27
Step 3: 123 × 45
Step 4: 77 × 88
Step 5: 92 × 63
Step 6: 53 × 105
Step 7: 314 × 206
Step 8: 113 × 104
Step 9: 525 × 413
Step 10: 678 × 345

Chapter 5

Base Method for Multiplication

The **Base Method** (also known as the **Nikhilam Sutra** in Vedic Math) is a useful technique in mental mathematics for multiplication when numbers are close to a base (such as 10, 100, 1000, etc.). Here are the steps:

Steps for multiplication using the Base Method

1. Select a base close to both numbers (10, 100, 1000, etc.). The base should be a power of 10.
2. Subtract base from each number to find the deviations (positive or negative).
3. Multiply the two deviations.
4. Add one number to the other number's deviation (or vice versa).
5. The final answer contains two parts.
   - The *left part* is the result of the cross addition (Step 4).
   - The *right part* is the result of the deviation multiplication (Step 3).
   - The number of digits in the right part must match the number of zeroes in the base.
   - Adjust using carry-over or borrowing if required.

The key to handling the base method efficiently is understanding how to adjust for carries. These adjustments ensure that the final result is correctly formatted according to the base used.

Rules for Carry Adjustments

1. The right part of final answer must have the same number of digits as the number of zeroes in base.
2. If the right part exceeds base, transfer the excess to the left part as a carry. Value of 1 carry will be equal to base.

3. If right part of final answer is negative, borrow carries from left part. Again, value of one carry will be equal to base.

Next, we illustrate the base method with some examples

Example 1: Multiply 102 × 103

Step 1: Choose base. Base $B = 100$

Step 2: Calculate deviations.

$$102 - 100 = +2, 103 - 100 = +3$$

Step 3: Multiply deviations.

$$2 \times 3 = 6$$

Step 4: Cross addition

$$102 + 3 = 105$$

Step 5: Combine results. Since the base is 100, the right part must contain **2 digits**. So we write the product of deviations as **06** (with leading zero if necessary).

$$105 \mid 06 = 10506$$

Thus

$$102 \times 103 = 10506.$$

Example 2: Multiply 89 × 91

Step 1: Choose base. Base $B = 100$

Step 2: Calculate deviations.

$$89 - 100 = -11, 91 - 100 = -9$$

Step 3: Multiply deviations.

$$(-11) \times (-9) = 99$$

Step 4: Cross addition

$$89 - 9 = 80$$

Step 5: Combine results. The base is 100, so the right part must have **2 digits**. Since 99 already has 2 digits, we write:

$$80 \mid 99 = 8099$$

Thus

$$89 \times 91 = 8099.$$

Example 3: Multiply 85 × 107

Step 1: Choose base. Base $B = 100$

Step 2: Calculate deviations.

$$85 - 100 = -15,\ 107 - 100 = +7$$

Step 3: Multiply deviations.

$$(-15) \times 7 = -105$$

Step 4: Cross addition

$$85 + 7 = 92$$

Step 5: Combine results. The base is 100, so the right part must have **2 digits**. We adjust the left part by borrowing 2 carries (i.e., 200):

$$92 - 2 = 90,\ -105 + 200 = 95 \Rightarrow 90 \mid 95 = 9095$$

Thus

$$85 \times 107 = 9095.$$

Example 4: Multiply 1020 × 991

Step 1: Choose base. Base $B = 1000$

Step 2: Calculate deviations.

$$1020 - 1000 = +20,\ 991 - 1000 = -9$$

Step 3: Multiply deviations.

$$20 \times (-9) = -180$$

Step 4: Cross addition

$$991 + 20 = 1011$$

Step 5: Combine results. The base is 1000, so the right part must have **3 digits**. We borrow 1 carry (1000) to adjust the negative result:

$$1011 - 1 = 1010,\ -180 + 1000 = 820 \Rightarrow 1010 \mid 820 = 1010820$$

Thus

$$1020 \times 991 = 1010820.$$

Example 5: Multiply 1023 × 1011

Step 1: Choose base. Base $B = 1000$

Step 2: Calculate deviations.

$$1023 - 1000 = +23,\ 1011 - 1000 = +11$$

Step 3: Multiply deviations.

$$23 \times 11 = 253$$

Step 4: Cross addition

$$1023 + 11 = 1034$$

Step 5: Combine results. Since the base is 1000, the right part must have **3 digits**. No adjustment needed, as 253 already has 3 digits:

$$1034 \mid 253 = 1034253$$

Thus

$$1023 \times 1011 = 1034253.$$

Example 6: Multiply 1019 × 989

Step 1: Choose base. Base $B = 1000$

Step 2: Calculate deviations.

$$1019 - 1000 = +19,\ 989 - 1000 = -11$$

Step 3: Multiply deviations.

$$19 \times (-11) = -209$$

Step 4: Cross addition

$$989 + 19 = 1008$$

Step 5: Combine results. The base is 1000, so the right part must have **3 digits**. We borrow 1 carry (1000) to adjust the negative result:

$$1008 - 1 = 1007, \quad -209 + 1000 = 791 \Rightarrow 1007 \mid 791 = 1007791$$

Thus

$$1019 \times 989 = 1007791.$$

Practice Exercises

Use the base method to solve the following:

1. 96×94
2. 105×108
3. 999×997
4. 115×112
5. 87×93
6. 95×78
7. 101×92
8. 1001×988
9. 1011×988
10. 985×978

Hint: Choose a suitable base, calculate deviations, multiply them, apply cross addition, and adjust the result as needed.

Chapter 6

Sub-base Method for Multiplication

The **Sub-Base Method**, also known as the **Nikhilam Sutra** in Vedic Mathematics, is a useful technique in mental mathematics for multiplication when numbers are close to a multiple of base (such as 10, 100, 1000, etc.). It builds upon the standard Base Method, offering enhanced applicability through the use of subbase.

Steps for Multiplication Using the Sub-Base Method

1. Choose a base B such that both numbers are close to a natural multiple of B i.e., $S \times B$. This multiple S is called the **sub-base**.
2. Calculate the deviations of both numbers from the sub-base:

$$\text{Deviation} = \text{Number} - (S \times B)$$

3. Multiply the deviations.
4. Add one number to the other's deviation (or vice versa), then multiply this sum by the sub-base S.
5. The final result has two parts:
   - The left part is the result from Step 4.
   - The right part is the product of the deviations from Step 3.
6. Adjust the right part to match the number of digits in the base, carrying over or borrowing as needed.

The key to handling the base method efficiently is understanding how to adjust for carries. These adjustments ensure that the final result is correctly formatted according to the base used.

Rules for Carry Adjustments

1. The right part of final answer must have the same number of digits as the number of zeroes in base.
2. If the right part exceeds base, transfer the excess to the left part as a carry. Value of 1 carry will be equal to base.

3. If right part of final answer is negative, borrow carries from left part. Again, value of one carry will be equal to base.
   Next, we illustrate the base method with some examples

Example 1: Multiplication of 32 × 37

Step 1: Choose base and sub-base. Base $B = 10$, Sub-base $S = 3$.

Step 2: Calculate deviations.

$$32 - (3 \times 10) = 2, 37 - (3 \times 10) = 7$$

Step 3: Multiply deviations.

$$2 \times 7 = 14$$

Step 4: Add one deviation to the other number, then multiply by sub-base.

$$(32 + 7) \times 3 = 117$$

Step 5: Combine results. Since the base is 10, the right part must be a single digit. We carry over 1 from 14 to the left.

$$117 \mid 14 = 1184$$

Final Answer:

$$1184$$

Example 2: Multiplication of 78 × 67

Step 1: Choose base and sub-base. Base $B = 10$, Sub-base $S = 7$.

Step 2: Calculate deviations.

$$78 - (7 \times 10) = 8, 67 - (7 \times 10) = -3$$

Step 3: Multiply deviations.

$$8 \times (-3) = -24$$

Step 4: Add one deviation to the other number, then multiply by sub-base.

$$(78 - 3) \times 7 = 525$$

Step 5: Combine results. We borrow 3 carries (i.e., 30) from 525.

$$525 \mid -24 = 5226$$

Final Answer:

$$5226$$

Example 3: Multiplication of 311 × 296

Step 1: Choose base and sub-base. Base $B = 100$, Sub-base $S = 3$.

Step 2: Calculate deviations.

$$311 - (3 \times 100) = 11, 296 - (3 \times 100) = -4$$

Step 3: Multiply deviations.

$$11 \times (-4) = -44$$

Step 4: Add one deviation to the other number, then multiply by sub-base.

$$(311 - 4) \times 3 = 921$$

Step 5: Combine results. We borrow 1 carry (i.e., 100) from the left

$$921 \mid -44 = 92056$$

Final Answer: 92056

Example 4: Multiplication of 4017 × 3979

Step 1: Choose base and sub-base. Base $B = 1000$, Sub-base $S =$

Step 2: Calculate deviations.

$4017 - (4 \times 1000) = 17,$ $3979 - (4 \times 1000) = -21$

Step 3: Multiply deviations.

$$17 \times (-21) = -357$$

Step 4: Add one deviation to the other number, then multiply sub-base.

$$(4017 - 21) \times 4 = 15984$$

Step 5: Combine results. We borrow 1 carry (i.e., 1000) from the

$$15984 \mid -357 = 15983643$$

Final Answer: 15983643

Example 5: Multiplication of 589 × 617

Step 1: Choose base and sub-base. Base $B = 100$, Sub-base S

Step 2: Calculate deviations.

$$589 - (6 \times 100) = -11,\ 617 - (6 \times 100) = 17$$

Step 3: Multiply deviations.

$$(-11) \times 17 = -187$$

Step 4: Add one deviation to the other number, then multiply b sub-base.

$$(589 + 17) \times 6 = 606 \times 6 = 3636$$

Step 5: Combine results. We borrow 2 carries (i.e., 200) from the left

$$3636 \mid -187 = 363413$$

Final Answer: 363413

Example 6: Multiplication of 9019 × 8989

Step 1: Choose base and sub-base. Base $B = 1000$, Sub-base $S = 9$

Step 2: Calculate deviations.

$$9019 - (9 \times 1000) = 19,\ 8989 - (9 \times 1000) = -11$$

Step 3: Multiply deviations.

$$19 \times (-11) = -209$$

Step 4: Add one deviation to the other number, then multiply by sub-base.

$$(9019 - 11) \times 9 = 9008 \times 9 = 81072$$

Step 5: Combine results. We borrow 1 carry (i.e., 1000) from the left

$$81072 \mid -209 = 81071791$$

Final Answer: 81071791

Exercises

Solve the following multiplications using the Sub Base Method:

1. 196 × 194
2. 67 × 51
3. 1999 × 1997
4. 215 × 212
5. 205 × 192
6. 125 × 123
7. 815 × 792
8. 2015 × 1992
9. 871 × 923
10. 315 × 307

Chapter 7

Yavadunum Method for Cubing

The **Yavadunum Method** is a useful technique in mental mathematics for finding cubes of numbers which are close to a multiple of base (such as 10, 100, 1000, etc.). Here are the steps:

Steps for multiplication using the Base Method

1. Suppose N be number whose cube is to be calculated.
2. Let B denotes the base (such as 10, 100, 1000, etc.). Base B is selected such that N is very close to a multiple it.
3. S denote the sub base, it is the multiple of B, which is close to N.
4. Calculate deviation $D = N - (S \times B)$.
5. The final answer consists of three parts and denoted as

$$(N + 2D)S^2 / 3D^2S / D^3$$

The final answer is obtained by matching the number of digits in middle part and right part as the number of zeroes in base, by adjusting carry.

The key to handling the Yavadunum method efficiently is understanding how to adjust for carries. These adjustments ensure that the final result is correctly formatted according to the base used.

Rules for Carry Adjustments

1. The right part and middle part of final answer must have the same number of digits as the number of zeroes in base.
2. If the right part or middle part exceeds base, transfer the excess to the instant left part in the final answer, as a carry. Value of 1 carry will be equal to base.

3. If right part of final answer is negative, borrow carries from middle part in final answer. Again, value of one carry will be equal to base.

Next, we illustrate the base method with some examples

Example 1: Find the Cube of 23

We use a method based on base approximation. Since 23 is close to 20, we write:

$$23 = 2 \times 10 + 3 \quad (\text{Here, } S = 2, B = 10, D = 3).$$

Step-by-Step Calculation

1. **Left Part:**

$$(N + 2D) \times S^2 = (23 + (2 \times 3)) \times 2^2 = 29 \times 4 = 116$$

2. **Middle Part:**

$$3D^2 \times S = 3 \times 3^2 \times 2 = 3 \times 9 \times 2 = 54$$

3. **Right Part:**

$$D^3 = 3^3 = 27$$

Carry Adjustment (Base 10)

1. Right Part: 27 → keep 7, carry 2 to the middle part.
2. Middle Part: 54 + 2 = 56 → keep 6, carry 5 to the left part.
3. Left Part: 116 + 5 = 121

Final Answer

$$\mathbf{23^3 = 12167}$$

Example 2: Find the Cube of 191

Since 191 is close to 200, we write:

$$191 = 2 \times 100 - 9 \ (\text{Here, } S = 2, B = 100, D = -9).$$

Step-by-Step Calculation

1. **Left Part:**

$$(N + 2D) \times S^2 = (191 + (2 \times (-9))) \times 2^2$$
$$= (191 - 18) \times 4 = 173 \times 4 = 692$$

2. **Middle Part:**

$$3D^2 \times S = 3 \times (-9)^2 \times 2 = 3 \times 81 \times 2 = 486$$

3. **Right Part:**

$$D^3 = (-9)^3 = -729$$

Carry Adjustment (Base 100)

1. Right Part: –729 → carry –8 to the middle part, add 800 to make it 71.
2. Middle Part: 486 – 8 = 478, keep 78, and shift 4 as carry to left part
3. Left Part: 692 + 4 = 696

Final Answer

$$\mathbf{191^3 = 6967871}$$

Example 3: Find the Cube of 83

Since 83 is close to 80, we write:

$$83 = 8 \times 10 + 3 \ (\text{Here, } S = 8, B = 10, D = 3).$$

Step-by-Step Calculation

1. **Left Part:**

$$(83 + (2 \times 3)) \times 8^2 = 89 \times 64 = 5696$$

2. **Middle Part:**

$$3 \times 3^2 \times 8 = 3 \times 9 \times 8 = 216$$

3. **Right Part:**

$$3^3 = 27$$

Carry Adjustment (Base 10)

1. Right Part: 27 → keep 7, carry 2 to middle.
2. Middle Part: 216 + 2 = 218 → keep 8, carry 21 to left.
3. Left Part: 5696 + 21 = 5717

Final Answer

$$\mathbf{83^3 = 571787}$$

Example 4: Find the Cube of 719

Since 719 is close to 700, we write:

$$719 = 7 \times 100 + 19 \quad (\text{Here, } S = 7, B = 100, D = 19).$$

Step-by-Step Calculation

1. **Left Part:**

$$(719 + (2 \times 19)) \times 7^2 = (719 + 38) \times 49 = 757 \times 49 = 37093$$

2. **Middle Part:**

$$3 \times 19^2 \times 7 = 3 \times 361 \times 7 = 7581$$

3. **Right Part:**

$$19^3 = 6859$$

Carry Adjustment (Base 100)

1. Right Part: 6859 → keep 59, carry 68 to middle.
2. Middle Part: 7581 + 68 = 7649 → keep 49, carry 76 to left.
3. Left Part: 37093 + 76 = 37169

Final Answer

$$\mathbf{719^3 = 371694959}$$

Example 5: Find the Cube of 591

Since 591 is close to 600, we write:

$$591 = 6 \times 100 - 9 \text{ (Here, } S = 6, B = 100, D = -9).$$

Step-by-Step Calculation

1. **Left Part:**

$$(591 + (2 \times (-9))) \times 6^2 = (591 - 18) \times 36 = 573 \times 36 = 20628$$

2. **Middle Part:**

$$3 \times (-9)^2 \times 6 = 3 \times 81 \times 6 = 1458$$

3. **Right Part:**

$$(-9)^3 = -729$$

Carry Adjustment (Base 100)

1. Right Part: −729 → add 800 to get 71, carry −8.
2. Middle Part: 1458 − 8 = 1450 → keep 50, carry 14 to left.
3. Left Part: 20628 + 14 = 20642

Final Answer

$$\mathbf{591^3 = 206425071}$$

Example 6: Find the Cube of 2011

Since 2011 is close to 2000, we write:

$$2011 = 20 \times 100 + 11 \text{ (Here, } S = 20, B = 100, D = 11).$$

Step-by-Step Calculation

1. **Left Part:**

$$(2011 + (2 \times 11)) \times 20^2 = 2033 \times 400 = 813200$$

2. **Middle Part:**

$$3 \times 11^2 \times 20 = 3 \times 121 \times 20 = 7260$$

3. **Right Part:**

$$11^3 = 1331$$

Carry Adjustment (Base 100)

1. **Right Part:** 1331 → keep 31, carry 13 to middle.
2. **Middle Part:** 7260 + 13 = 7273 → keep 73, carry 72 to left.
3. **Left Part:** 813200 + 72 = 813272

Final Answer

$$2011^3 = 8132727331$$

Exercises

Solve the following multiplications using the Yavadunum Method:

1. 196
2. 51
3. 199
4. 212
5. 192
6. 123
7. 815
8. 201
9. 923
10. 307

Chapter 8

Square Roots of Exact Squares

To find the square root of exact squares, we must first determi[ne] whether the given number is a perfect square. We can verify th[is] using the following fundamental rules:

1. A perfect square ends in 0, 1, 4, 5, 6, or 9.
2. A number is **not** a perfect square if it ends in 2, 3, 7, or 8.
3. The number should end in an even number of zeros.
4. If the number ends in 6, then its second-last digit should b[e] odd.
5. If the number does **not** end in 6, then its second-last digit shoul[d] be even.
6. If the number is even, its last two digits should be divisible by [...]
7. If a given number has n digits:
   - If n is even, its square root will have $n/2$ digits.
   - If n is odd, its square root will have $(n + 1)/2$ digits.

Squares of the First 10 Natural Numbers

Table 8.1: Squares of the first 10 natural numbers

Number	*Square*	*Last Digit of Square*
1	1	1
2	4	4
3	9	9
4	16	6
5	25	5
6	36	6
7	49	9
8	64	4
9	81	1
10	100	0

From the above table, we can conclude that:

1. If the square ends in 1, then the last digit of its square root will either be 1 or 9.
2. If the square ends in 4, then the last digit of its square root will either be 2 or 8.
3. If the square ends in 5, then the last digit of its square root will be 5.
4. If the square ends in 6, then the last digit of its square root will either be 4 or 6.
5. If the square ends in 9, then the last digit of its square root will either be 3 or 7.

Rules for Finding Square Roots of Perfect Squares

1. Make groups of two digits in the number, starting from the right.
2. Determine the last digit of the square root by observing the last digit of the given number.
3. Determine the first digit of the square root by analyzing the first group of digits.
4. We find the square of the number ending in 5 using the sutra *"Ekadhikena Purvena"* between our two possible answers.
5. If our given number is greater or smaller than this square, we can choose the correct answer from the two options accordingly.

This method is works when the given number is a perfect square and has more than five digits, this method is less suitable. Let us understand it with a few examples. The numbers considered in these examples are prefect squares.

Example 1: Find the square root of 1024

Step 1: We make groups of 2 digits starting from the right. The number is 1024, so we have 2 groups: (10, 24).

Step 2: Since the square ends in 4, the square root must end in either 2 or 8 (by observing the table of squares). Thus, we have two possible options for the unit digit of our answer.

Step 3: Observing the leftmost group (10): Since $3^2 = 9$ and $4^2 = 16$, the number 10 lies between 9 and 16. Thus, the first digit of our square root will be 3. We cannot take 4 because $40^2 = 1600$ is greater than 1024. So, we take 3 as the first digit of our answer.

Now, our options are 32 and 38.

Step 4: Using the sutra *"Ekadhikena Purvena"*:

$$35^2 = 1225$$

Since 1024 is an exact square and is less than 1225, the square root must be less than 35. Thus, the correct answer is:

$$\sqrt{1024} = 32$$

Example 2: Find square root of 2116

Step 1: We make groups of 2 digits starting from the right. Here we have two groups: (21, 16).

Step 2: Since the square ends in 6, the square root must end in either 4 or 6.

Step 3: Observe the left group number 21: Since $4^2 = 16$ and $5^2 = 25$ the number 21 lies between these two squares. Thus, the first digit of our square root will be 4.

We cannot take 5 because $50^2 = 2500$, which is greater than 2116. So, we take 4 as the first digit of our answer.

Now, our options are 44 and 46.

Step 4: Using the sutra *"Ekadhikena Purvena"*:

$$45^2 = 2025$$

Since 2116 > 2025 and 2116 is a perfect square, the square root must be more than 45. Thus, the correct answer is:

$$\sqrt{2116} = 46$$

Example 3: Find square root of 17689

Step 1: We make groups of 2 digits starting from the right. Here we have: (176, 89)

Step 2: Since the square ends in 9, the square root must end in either 3 or 7.

Step 3: Observe the left group number 176: Since $13^2 = 169$ and $14^2 = 196$, the number 176 lies between these two squares. Thus, the first digits of our square root will be 13.

Now our options are 133 and 137.

Step 4: We know that:

$$135^2 = 18225$$

Since 17689 < 18225, the square root must be less than 135. Thus, the correct answer is:

$$\sqrt{17689} = 133$$

Example 4: Find square root of 12769

Step 1: We make groups of 2 digits starting from the right. Here we have: (127, 69)

Step 2: The number ends in 9, so the square root must end in either 3 or 7.

Step 3: Look at the left group, 127:

$$11^2 = 121 \quad \text{and} \quad 12^2 = 144$$

Since 127 lies between these two squares, the first digits will be 11. We cannot take 12, because $120^2 = 14400 > 12769$. Now our options are 113 and 117.

Step 4: Using the sutra:

$$115^2 = 13225$$

Since 12769 < 13225, the square root must be less than 115. Thus, the correct answer is:

$$\sqrt{12769} = 113$$

Example 5: Find square root of 11881

Step 1: We make groups of 2 digits starting from the right. Here we have: (118, 81)

Step 2: Since the square ends in 1, the square root must end in either 1 or 9.

Step 3: Look at the left group, 118:

$$10^2 = 100 \quad \text{and} \quad 11^2 = 121$$

Since 118 lies between these, the first digits will be 10. Now our options are 101 and 109.

Step 4: Using the sutra:

$$105^2 = 11025$$

Since 11881 > 11025, and it's a perfect square, the correct answer is:

$$\sqrt{11881} = 109$$

Practice Problems

Find the square roots of the following perfect squares by mere observation:

1. $\sqrt{529}$
2. $\sqrt{1089}$
3. $\sqrt{2401}$
4. $\sqrt{6724}$
5. $\sqrt{3969}$
6. $\sqrt{9025}$
7. $\sqrt{12321}$
8. $\sqrt{12544}$
9. $\sqrt{17161}$
10. $\sqrt{11881}$

Chapter 9

Cube Roots of Exact Cubes

Cube roots of exact cubes can be found using a pattern similar to that of square roots. Let us first study the cubes of the first natural numbers to determine the last digit of the cubes.

Table 9.1: Cubes of the first nine natural numbers

Number	*Cube*	*Last Digit in Cube*
1	1	1
2	8	8
3	27	7
4	64	4
5	125	5
6	216	6
7	343	3
8	512	2
9	729	9

1. If the cube ends in 1, then the cube root ends in 1.
2. If the cube ends in 2, then the cube root ends in 8.
3. If the cube ends in 3, then the cube root ends in 7.
4. If the cube ends in 4, then the cube root ends in 4.
5. If the cube ends in 5, then the cube root ends in 5.
6. If the cube ends in 6, then the cube root ends in 6.
7. If the cube ends in 7, then the cube root ends in 3.
8. If the cube ends in 8, then the cube root ends in 2.
9. If the cube ends in 9, then the cube root ends in 9.
10. If the cube ends in 0, then the cube root ends in 0.

We observe that there is no overlapping of digits in the cube of the first ten natural numbers, unlike in the case of squares.

Rules for Finding Cube Roots

1. Make groups of two digits in the number, starting from the right.
2. Determine the last digit of the square root by observing the last digit of the given number.
3. Determine the first digit of the square root by analyzing the first group of digits.
4. We find the square of the number ending in 5 using the sutra *"Ekadhikena Purvena"* between our two possible answers.
5. If our given number is greater or smaller than this square, we can choose the correct answer from the two options accordingly.

Note: This method is effective when:
1. The given number is a perfect cube.
2. The number has atmost five to six digits.

If the cube has more than five digits, this method is less suitable. Let us understand it with a few examples. The numbers considered in these examples are prefect squares.

Examples

Example 1: Find the cube root of 1728

Step 1—Group the digits: We divide the number into groups of three digits from the right: (1, 728). There are 2 groups, so the cube root must have 2 digits.

Step 2—Determine the unit digit: The number ends in 8. We observe:

$$2^3 = 8$$

Hence, the unit digit of the cube root is 2.

Step 3—Estimate the leading digit: The first group is 1. We find that:

$$1^3 = 1 \quad \text{and} \quad 2^3 = 8$$

Since 1 lies between 1^3 and 2^3, the tens digit is 1.

Final Answer:

$$\sqrt[3]{1728} = 12$$

Example 2: Find the cube root of 13824

Step 1—Group the digits: (13, 824) — the cube root has 2 digits.

Step 2—Unit digit analysis: The number ends in 4. We recall:

$$4^3 = 64$$

So, the cube root ends in 4.

Step 3—Identify the leading digit: The left group is 13.

$$2^3 = 8 \quad \text{and} \quad 3^3 = 27$$

Since 13 lies between 8 and 27, the leading digit is 2.

Final Answer:

$$\sqrt[3]{13824} = 24$$

Example 3: Find the cube root of 175616

Step 1—Group the digits: (175, 616)

Step 2—Observe the last digit: The number ends in 6.

$$6^3 = 216$$

So, the unit digit of the cube root is 6.

Step 3: Determine the first digit: We consider 175:

$$5^3 = 125 \quad \text{and} \quad 6^3 = 216$$

Since 175 lies between these two, the leading digit is 5.

Final Answer:

$$\sqrt[3]{175616} = 56$$

Example 4: Find the cube root of 681472

Step 1—Group the digits: (681, 472)

Step 2—Last digit analysis: The number ends in 2.

$$8^3 = 512$$

Thus, the unit digit of the cube root is 8.

Step 3—Estimate the leading digit: Observe 681:

$$8^3 = 512 \quad \text{and} \quad 9^3 = 729$$

Since 681 lies between 512 and 729, the tens digit is 8.

Final Answer:

$\sqrt[3]{681472} = 88$

These methods rely on careful digit grouping, pattern recognition in cube values, and mental approximation. They are efficient for mental math and useful for developing a deeper understanding of number properties, as emphasized in ancient Bharatiya and classical mathematical approaches.

Exercise

Find the cube roots of the following exact cubes:

1. $\sqrt[3]{4913}$
2. $\sqrt[3]{74088}$
3. $\sqrt[3]{456533}$
4. $\sqrt[3]{3176523}$
5. $\sqrt[3]{1030301}$
6. $\sqrt[3]{195112}$
7. $\sqrt[3]{1092727}$

Answers to Exercises

1. $\sqrt[3]{4913} = 17$
2. $\sqrt[3]{74088} = 42$
3. $\sqrt[3]{456533} = 77$
4. $\sqrt[3]{3176523} = 147$
5. $\sqrt[3]{1030301} = 101$
6. $\sqrt[3]{195112} = 58$
7. $\sqrt[3]{1092727} = 103$

Chapter 10

Divisibility

Most numbers can be divided by other numbers. The number 1 is different because it can only be divided by itself, so it is called indivisible. If a number has only two divisors, 1 and itself, it is called a prime number. If it has more than two divisors, it is called a composite number.

Now, let us explore how to determine the divisibility of different numbers.

Divisibility by 2

Any number that ends with 0, 2, 4, 6, or 8 is divisible by 2, as these digits indicate the number is even.

Examples: 37564, 4566482, and 678740278 are all even numbers, so they are divisible by 2.

Divisibility by 5

Numbers ending in 0 or 5 are divisible by 5.

Examples: 4985, 9263005, 54675, and 834675 all end with 0 or 5, making them divisible by 5.

Divisibility by 10

All numbers that end in 0 are divisible by 10.

Example: 76850, 550500, 498070 are completely divisible by 10.

Divisibility by 3

To determine whether a number is divisible by 3, calculate the sum of all its digits. If this total is itself divisible by 3, then the original number can be exactly divided by 3 without leaving a remainder.

This method provides a quick way to verify divisibility without performing full division.

Example: Determine if 562984 is divisible by 3.

First, find the sum of its digits:

$$5 + 6 + 2 + 9 + 8 + 3 = 33 \quad (10.1)$$

Since 33 is not divisible by 3, the number 552984 is **not** divisible by 3.

Example: Determine if 52073648 is divisible by 3.

Calculate the sum of the digits:

$$5 + 2 + 0 + 7 + 3 + 6 + 4 + 8 = 35 \quad (10.2)$$

Because 35 is divisible by 3, the number 52073648 is **divisible** by 3.

Divisibility by 9

A number is divisible by 9 if the sum of its digits results in a number divisible by 9. To check this, add all the digits together; if this sum is divisible by 9, the entire number is divisible by 9.

An alternative and efficient technique is the method known as casting out 9s. This involves eliminating all 9s and any combinations of digits that sum to 9 from the number. After removing these, add the remaining digits and continue until only a single digit is left. If this digit is 0 or 9, the original number is divisible by 9.

Example: Is 8327451 divisible by 9?

Remove all 9s and any groups of digits that add up to 9, such as (8+1), (7+2). After this, the leftover digit is 4, which is not 0 or 9. Hence, 8327451 is **not** divisible by 9.

Example: Is 9241836 divisible by 9?

Eliminate all 9s and groups of digits summing to 9, such as the digit 9 itself and (2+7), (1+8). This leaves 0, so 9241836 is **divisible** by 9.

Example: Is 5384163 divisible by 9?

Remove all 9s and any groups of digits which sum to 9, like (5+4), (3+6). The remaining digit is 3, which is neither 0 nor 9. Therefore, 5384163 is **not** divisible by 9.

Divisibility by 4

A number is divisible by 4 if the last two digits form a number that is divisible by 4. Alternatively, you can add the last digit to twice the second-last digit; if this sum is divisible by 4, then the entire number is divisible by 4.

Example: Is 583921 divisible by 4?

Calculate the sum of the last digit and twice the second-last digit:

$$1 + 2 \times 2 = 5$$

Since 5 is not divisible by 4, 583921 is **not** divisible by 4.

Example: Is 76248 divisible by 4?

Calculate the sum of the last digit and twice the second-last digit:

$$8 + 2 \times 4 = 16$$

Since 16 is divisible by 4, 76248 is **divisible** by 4.

Example: Is 135612 divisible by 4?

Calculate the sum of the last digit and twice the second-last digit:

$$2 + 2 \times 1 = 4$$

Because 4 is divisible by 4, 135612 is **divisible** by 4.

Divisibility by 8

A number is divisible by 8 if the value formed by its last three digits is divisible by 8. Another method involves taking the last digit, adding twice the second-last digit, and four times the third-last digit; if the total sum is divisible by 8, then the entire number is divisible by 8.

Example: Determine whether 74638 is divisible by 8.

Calculate the sum:

$$8 + 2 \times 3 + 4 \times 6 = 8 + 6 + 24 = 38$$

Since 38 is not divisible by 8, the number 74638 is **not divisible** by 8.

Example: Check if 159264 is divisible by 8.

Compute the sum:

$$4 + 2 \times 6 + 4 \times 2 = 4 + 12 + 8 = 24$$

Because 24 is divisible by 8, the number 159264 is **divisible** by 8.

Divisibility by 11

A number is divisible by 11 if the difference between the sum of the digits in its odd positions and the sum of the digits in its even positions is either 0 or a multiple of 11.

Example: Check whether 9263745 is divisible by 11.

Identify the digits in odd and even positions (counting positions from the rightmost digit): Odd-positioned digits: 5, 3, 6, 9
Even-positioned digits: 4, 7, 2
Calculate the difference:

$$5 + 3 + 6 + 9 = 23$$

$$4 + 7 + 2 = 13$$

$$23 - 13 = 10$$

Since 10 is neither 0 nor a multiple of 11, the number 9263745 is **not** divisible by 11.

Example: Check whether 48123651 is divisible by 11.

Odd-positioned digits: 1, 3, 2, 4
Even-positioned digits: 5, 6, 8
Difference:

$$1 + 3 + 2 + 4 = 10$$

$$5 + 6 + 8 = 19$$

$$19 - 10 = 9$$

Since 9 is not 0 or a multiple of 11, 48123651 is **not** divisible by 11.

Example: Determine if 709278462 is divisible by 11.

Odd-positioned digits: 2, 8, 7, 0, 7

$$2 + 8 + 7 + 0 + 7 = 24$$

Even-positioned digits: 6, 4, 9, 2
Difference:

$$6 + 4 + 9 + 2 = 21$$

$$24 - 21 = 3$$

Since 3 is neither 0 nor a multiple of 11, the number 709278462 is not divisible by 11.

Divisibility by Prime Numbers

To determine whether a number is divisible by a prime number, one can use its associated **Ekadhika** value. This method involves the following steps:

1. Multiply the last digit of the number by the Ekadhika of the prime number.
2. Add this product to the remaining part of the number obtained by removing the last digit.
3. Repeat this process with the new number until the result is a number whose divisibility by the prime number is already known.

If at any point the resulting number is divisible by the prime number, then the original number is also divisible by that prime.

How to find Ekadhika: The Ekadhika of a prime number is found by multiplying it with a certain multiplier such that the product ends with the digit 9. The Ekadhika is then obtained by adding one to the digit(s) before 9 in that product.

For example:

Ekadhika of 7 : $7 \times 7 = 49 \Rightarrow 4 + 1 = 5$
Ekadhika of 13 : $13 \times 3 = 39 \Rightarrow 3 + 1 = 4$
Ekadhika of 19 : $19 \times 1 = 19 \Rightarrow 1 + 1 = 2$ Thus, the Ekadhika values are:

$$7 \to 5, \quad 13 \to 4, \quad 19 \to 2$$

This method simplifies divisibility testing by reducing large numbers step-by-step using the Ekadhika multiplier.

Divisibility by 7

Now we use Ekadhika to check divisibility by 7. The Ekadhika of 7 is 5. We multiply the last digit of the number by its Ekadhika and add it to the other digits before the last digit. We keep doing this until we reach a number that we already know is divisible by 7.

Example: Is 743693 divisible by 7?

Note that, Ekadhika of 7 is 5.

Last digit = 3, $3 \times 5 = 15$, $74369 + 15 = 74384$
Last digit = 4, $4 \times 5 = 20$, $7438 + 20 = 7458$
Last digit = 8, $8 \times 5 = 40$, $745 + 40 = 785$
Last digit = 5, $5 \times 5 = 25$, $78 + 25 = 103$
Last digit = 3, $3 \times 5 = 15$, $10 + 15 = 25$

Since 25 is **not** divisible by 7, the number 743693 is **NOT** divisible by 7.

Divisibility by 13

Example: Is 126594 divisible by 13?

Note that, Ekadhika of 13 is calculated as:

$$13 \times 3 = 39, \quad 3 + 1 = 4$$

Last digit is 4, so

$$4 \times 4 = 16, 12659 + 16 = 12675$$

Last digit now is 5, so

$$5 \times 4 = 20, 1267 + 20 = 1287$$

Last digit now is 7, so

$$7 \times 4 = 28, 128 + 28 = 156$$

Last digit now is 6, so

$$6 \times 4 = 24, 15 + 24 = 39$$

Since 39 is divisible by 13, the number 126594 is **completely** divisible by 13.

Divisibility Rules for Composite Numbers

To check the divisibility of composite (non-prime) numbers, we combine the divisibility rules of their factors. For example, 6 has its factors as 2 and 3, so for a number to be divisible by 6, it must be divisible by both 2 and 3.

Some Rules

1. A number is divisible by 6 if it is divisible by both 2 and 3.
2. A number is divisible by 12 if it is divisible by both 3 and 4.
3. A number is divisible by 15 if it is divisible by both 3 and 5.
4. A number is divisible by 18 if it is divisible by both 2 and 9.
5. A number is divisible by 24 if it is divisible by both 3 and 8.
6. A number is divisible by 30 if it is divisible by both 3 and 10.
7. A number is divisible by 20 if it ends in zero and the second last digit is even.

Example: Is 1379112 divisible by 24?

If a number is divisible by both 3 and 8, then it is divisible by 24.

To check divisibility by 3, we add all the digits:

$$1 + 3 + 7 + 9 + 1 + 1 + 2 = 24, 2 + 4 = 6$$

Since 6 is divisible by 3, the number 1379112 is divisible by 3.

To check divisibility by 8, we add:

(last digit) + 2 × (second last digit) + 4 × (third last digit)

To check the divisibility by 8, we add the last digit plus twice the second-last digit plus four times the third-last digit:

$$2 + (2 \times 1) + (4 \times 1) = 8$$

So 1379112 is divisible by 8. Since 1379112 is divisible by both 3 and 8, it is divisible by 24.

Exercise

Which of the following are divisible by the given number?

1. 4321, 7865, 6745, 63963 by 3
2. 536, 8765, 48654, 4864 by 5
3. 540, 5642, 78546, 64678 by 6
4. 5436, 79492, 78654, 65846 by 7

5. 3246, 5368, 2178, 87321 by 8
6. 76539, 95346, 476325, 7643254 by 9
7. 675334, 56487, 39765, 648768 by 11
8. 9870, 12345, 6545, 987080 by 15
9. 876, 187632, 756432, 76546 by 18
10. 4624, 875432, 841128, 78698 by 24

Chapter 11

Multiplication of Numbers to a Number only Consisting of 9's

This method takes advantage of the fact that such multipliers are exactly one less than a power of 10. In other words, if the multiplier has n digits (all 9's), then it can be written as $10^n - 1$. This method is applicable when number being multiplied with $10^n - 1$, have digits less than or equal to n.

Steps for multiplication with a number only consisting of 9's

Let N be the multiplicand and the multiplier be $10^n - 1$. Then result is split into two parts:

1. **Left-hand part (LHS):** Subtract 1 from the multiplicand:

$$\text{LHS} = N - 1.$$

2. **Right-hand part (RHS):** Subtract the multiplicand from the base 10^n:

$$\text{RHS} = 10^n - N = (10^n - 1) - (N - 1)$$

Note that once once left hand side term $N-1$ is written, the term $(10^n - 1) - (N - 1)$ can be written by keeping the sum of corresponding digits equal to 9.

The final product is obtained by writing the left-hand part followed by the right-hand part.

Example 1: Multiplying 15 × 99

Here, the multiplicand is $N = 15$ and the multiplier is 99.

1. **Left-hand part:**

$$15 - 1 = 14.$$

2. **Right-hand part:**

$$85 \text{ (since } 1 + 8 = 9 \text{ and } 4 + 5 = 9)$$

Thus, the product is:

$$15 \times 99 = 1485.$$

Example 2: Multiplying 587 × 999

For this example, $N = 587$ and the multiplier is 999 (which has 3 digits).

1. **Left-hand part:**

$$587 - 1 = 586.$$

2. **Right-hand part:**

413 (since $5 + 4 = 9$, $8 + 1 = 9$ and $6 + 3 = 9$)

So, the final product is:

$$587 \times 999 = 586\,413.$$

Example 3: Multiplying 123 × 9999

Here, the multiplicand is $N = 0123$ and the multiplier is 9999 (which has 4 digits).

1. **Left-hand part:**

$$0123 - 1 = 0122.$$

2. **Right-hand part:**

9877 (since $0 + 9 = 9$, $1 + 8 = 9$, $2 + 7 = 9$ and $2 + 7 = 9$)

So, the final product is:

$$123 \times 9999 = 1229877.$$

Example 4: Multiplying 1406 × 999999

Here, the multiplicand is $N = 001406$ and the multiplier is 999999 (which has 6 digits).

1. **Left-hand part:**

$$001406 - 1 = 001405.$$

2. **Right-hand part:**

998594 (since $0 + 9 = 9$, $0 + 9 = 9$, $1 + 8 = 9$, $4 + 5 = 9$, $0 + 9 = 9$ and $6 + 3 = 9$)

Thus, the product is:

$$1406 \times 999999 = 1405998594.$$

Example 5: Multiplying 6719001 × 9999999

Here, the multiplicand is $N = 6719001$ and the multiplier is 9999999 (which has 7 digits).

1. **Left-hand part:**

$$6719001 - 1 = 6719000.$$

2. **Right-hand part:**

3280999 (since $6 + 3 = 9$, $7 + 2 = 9$, $1 + 8 = 9$, $9 + 0 = 9$, $0 + 9 = 9$, $0 + 9 = 9$, $1 + 8 = 9$)

So, the final product is:

$$6719001 \times 9999999 = 67190003280999.$$

Exercises

Multiply the following using the method described above:

1. 47 × 99
2. 206 × 999
3. 1503 × 9999
4. 89 × 99
5. 712 × 999
6. 5301 × 9999
7. 84 × 99
8. 999 × 999
9. 67005 × 99999
10. 891002 × 999999

Hint: For each question, subtract 1 from the multiplicanc get the left-hand part. Then subtract the left-hand part from multiplier (or ensure each digit pair sums to 9) to get the right-hı part.

Chapter 12

Subtraction Using the Vedic Method

Vedic Mathematics is an ancient system of calculation that comes from the Vedas—Bharat's old and sacred books. One of the most interesting and useful tricks in Vedic Mathematics is subtraction using **complements**. Instead of doing long and difficult borrowing, we can subtract numbers quickly using the method called:

"Nikhilam Navataścaramam Daśatah."

(All from 9 and the last from 10)

This rule helps us subtract numbers in a much easier way by changing the subtraction into addition. This is very useful, especially for large numbers and for doing mental math.

How It Works

Let us say we want to subtract two numbers:

Let A be the bigger number (minuend) and
Let B be the smaller number (subtrahend).
We follow these steps:

1. **Choose the Base:** Use a base like 10, 100, 1000, etc.—choose the smallest power of 10 that is bigger than B.
2. **Find the Complement of B:**
   - Subtract each digit from 9, except the last digit.
   - Subtract the last digit from 10.
3. **Add the Complement to A.**
4. **Subtract the Base from the Result.**
   This gives us the final answer.

Example 1: Subtract 4678 from 8325

We want to calculate:

$$8325 - 4678$$

Step 1: Base
Since 4678 is a 4-digit number, we choose the base 10000.

Step 2: Find the Complement of 4678
Using the rule "All from 9 and the last from 10":

$$9 - 4 = 5$$
$$9 - 6 = 3$$
$$9 - 7 = 2$$
$$10 - 8 = 2$$

So the complement is: 5322

Step 3: Add the Complement to 8325

$$8325 + 5322 = 13647$$

Step 4: Subtract the Base

$$13647 - 10000 = 3647$$

Therefore,

$$8325 - 4678 = 3647$$

Example 2: Subtract 3891 from 7250

Now we subtract:

$$7250 - 3891$$

Step 1: Base
Use base 10000.

Step 2: Complement of 3891

$$9 - 3 = 6$$
$$9 - 8 = 1$$
$$9 - 9 = 0$$
$$10 - 1 = 9$$

Complement = 6109

Step 3: Add to Minuend

$$7250 + 6109 = 13359$$

Step 4: Subtract the Base

$$13359 - 10000 = 3359$$

So,

$$7250 - 3891 = 3359$$

Conclusion

This method is quick, easy to learn, and fun to use. It helps students do subtraction faster without getting confused by borrowing. With practice, it can even be done in your head!